

The magnitude of the non-adiabatic pressure in the cosmic fluid

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ABSTRACT

Understanding the non-adiabatic pressure, or relative entropy, perturbation is crucial for studies of early-universe vorticity and Cosmic Microwave Background observations. We calculate the evolution of the linear non-adiabatic pressure perturbation from radiation domination to late times, numerically solving the linear governing equations for a wide range of wavenumbers. Using adiabatic initial conditions consistent with WMAP seven year data, we find nevertheless that the non-adiabatic pressure perturbation is non-zero and grows at early times, peaking around the epoch of matter/radiation equality and decaying in matter domination. At early times or large redshifts ($z = 10,000$) its power spectrum peaks at a comoving wavenumber $k \approx 0.2h/\text{Mpc}$, while at late times ($z = 500$) it peaks at $k \approx 0.02h/\text{Mpc}$.

Key words: Cosmology: theory

1 INTRODUCTION

Our understanding of the physics of the universe is improving steadily. Recent observational successes, in particular Cosmic Microwave Background (CMB) experiments and Large Scale Structure (LSS) surveys (Komatsu et al. 2011a; Ade et al. 2011; Abazajian et al. 2009), strongly favour the cosmological standard model, in which the anisotropies of the CMB and the LSS are sourced by quantum fluctuations formed during a period of accelerated expansion in the early universe – inflation (Liddle & Lyth 2000).

Since the decay of the scalar field(s) driving inflation into the standard matter fields is not fully understood, the power spectrum of the fluctuations in the scalar field is mapped onto the spectrum of the comoving curvature perturbation, \mathcal{R} , at horizon crossing. Since \mathcal{R} is conserved on large scales for adiabatic perturbations, the primordial power spectrum is not affected by the small scale physics, and can be used to set the initial conditions for Boltzmann codes (Bertschinger 1995; Seljak & Zaldarriaga 1996; Lewis et al. 2000; Doran 2005; Lesgourgues 2011). The current data allow for a small non-adiabatic contribution to the primordial perturbations, only providing an upper bound (Komatsu et al. 2011b).

This contribution is described by the non-adiabatic pressure, or entropy, perturbation, δP_{nad} . To date this has been calculated mainly to get a handle on the evolution of the curvature perturbation (Wands et al. 2000), and hence the primordial power spectrum, and also played a key role in characterising the initial conditions (“adiabatic” versus “isocurvature” (Liddle & Lyth 2000)). Observations of the CMB are fully consistent with purely adiabatic initial conditions (Komatsu et al. 2011b), although a level of primordial isocurvature can be allowed (references (Valiviita & Giannantonio 2009; Mangilli et al. 2010; Li et al. 2011) provide an non-exhaustive example of recent studies).

However, after the initial conditions have been imposed, there are no restrictions on the non-adiabatic contribution. In this letter we focus on the relative entropy perturbation, which naturally arises in any multi-component system. This contribution to δP_{nad} has so far not been studied in detail in realistic models, and we calculate its evolution numerically, starting from adiabatic initial conditions in agreement with the WMAP7 concordance model.

A non-zero entropy perturbation sources the evolution of the curvature perturbation, which will not be conserved on large scales beyond the time when the initial conditions are set. In practice, this will affect any calculation assuming conservation of the curvature perturbation after $z \sim 100,000$. Furthermore, it has been shown that the non-adiabatic pressure perturbation can generate vorticity in the early universe (Christopherson et al. 2009). Although this vorticity is generated at second order in perturbation theory, it might act as foreground and contribute to the primordial B-mode polarisation signal in the CMB on small scales.

2 DEFINITIONS

We consider linear, scalar perturbations to a flat Friedmann-Robertson-Walker (FRW) spacetime, following Malik & Wands (2005). The total density and the total pressure including background and perturbations, ρ and P , respectively, are related to the density and pressure of the component fluids by $\rho = \sum_{\alpha} \rho_{\alpha}$, $P = \sum_{\alpha} P_{\alpha}$, where Greek letters label the individual fluids.

The total pressure perturbation can be split into an adiabatic and non-adiabatic part $\delta P \equiv \delta P_{\text{nad}} + c_s^2 \delta \rho$, where c_s^2 is the adiabatic speed of sound, $c_s^2 \equiv \dot{P}/\dot{\rho}$, and an overdot denotes a derivative with respect to conformal time, η .

In the presence of more than one fluid, the total non-adiabatic pressure perturbation, δP_{nad} , may be further decomposed (Kodama & Sasaki 1984) as $\delta P_{\text{nad}} \equiv \delta P_{\text{intr}} + \delta P_{\text{rel}}$. The first part is the sum of the intrinsic entropy perturbation of each fluid, which vanishes for barotropic fluids. We assume here for simplicity that all individual fluids have a constant equation of state and are hence barotropic.

The second part of the non-adiabatic pressure perturbation is due to the *relative entropy perturbation* $S_{\alpha\beta}$ between different fluids, defined as

$$S_{\alpha\beta} \equiv -3\mathcal{H} \left(\frac{\delta\rho_\alpha}{\dot{\rho}_\alpha} - \frac{\delta\rho_\beta}{\dot{\rho}_\beta} \right), \quad (1)$$

where \mathcal{H} is the conformal Hubble parameter, $\mathcal{H} \equiv \dot{a}/a$. The relative non-adiabatic pressure perturbation between multiple fluids is then

$$\begin{aligned} \delta P_{\text{rel}} &= -\frac{1}{6\mathcal{H}\dot{\rho}} \sum_{\alpha,\beta} \dot{\rho}_\alpha \dot{\rho}_\beta (c_\alpha^2 - c_\beta^2) S_{\alpha\beta} \\ &= \frac{1}{2\dot{\rho}} \sum_{\alpha,\beta} (c_\alpha^2 - c_\beta^2) (\dot{\rho}_\beta \delta\rho_\alpha - \dot{\rho}_\alpha \delta\rho_\beta), \end{aligned} \quad (2)$$

with $c_\alpha^2 \equiv \dot{P}_\alpha/\dot{\rho}_\alpha$ the adiabatic sound speed of each individual fluid. It is straightforward to see that the non-adiabatic pressure perturbation is invariant under a gauge transformation. This also implies that if it is non-zero in one gauge, it will be non-zero in any other gauge; i.e. it can not be “gauged away”.

By definition, the relative non-adiabatic pressure perturbation does not contribute to the individual fluid evolution equations and instead enters the Einstein equations. It is perhaps in this context that the non-adiabatic pressure perturbation is most familiar, governing the evolution of the curvature perturbation on uniform density hypersurfaces, ζ , on large scales (Wands et al. 2000).

Boltzmann codes typically avoid the Einstein equations in which the non-adiabatic pressure perturbation would enter – these have instead been used as consistency checks – and so the system we are evolving remains closed at linear order.

However, at higher orders in perturbation theory, knowledge of the behaviour δP_{rel} becomes important. In particular, the non-adiabatic pressure produced by linear perturbations will have a significant impact on the curvature perturbation on uniform density hypersurfaces, ζ , at a non-linear level. A first study in this direction was undertaken at second order by Malik (2005), showing the dependence of ζ_2 on the relative entropy perturbations. It has also been shown that the non-adiabatic pressure perturbation can generate vorticity in the early universe (Christopherson et al. 2009). Furthermore, if a modification to a Boltzmann code includes a perturbed total pressure the non-adiabatic pressure perturbation should be included.

3 DYNAMICS AND INITIAL CONDITIONS

The concordance cosmology contains five fluids: baryonic matter, cold dark matter, photons, neutrinos and some form of dark energy. We assume the neutrinos to be massless. Defining the equation of state parameter $w_\alpha = P_\alpha/\rho_\alpha$, baryons (b) and CDM (c) have $w_b = w_c = c_b^2 = c_c^2 = 0$. Photons (γ) and neutrinos (ν) are relativistic species and have $w_\gamma = w_\nu = c_\gamma^2 = c_\nu^2 = 1/3$. Photons and baryons are coupled together by Compton scattering, while neutrinos stream freely. We employ a flat Λ CDM cosmology

consistent with the WMAP 7-year results (Komatsu et al. 2011b) with the parameters

$$\begin{aligned} \Omega_b h^2 &= 2.253 \times 10^{-2}, \quad \Omega_c h^2 = 0.112, \\ \Omega_\Lambda &= 0.728, \quad h = 0.704. \end{aligned} \quad (3)$$

Although we employ a cosmological constant, our approach also allows for a dark energy with $w_\phi \neq 1$ and therefore dark energy density perturbations. These would contribute to the non-adiabatic pressure perturbation at late times. Since our focus is on the early universe where dark energy perturbations are expected to contribute negligibly, results from Λ CDM will be very similar to those from more general models. Of course, formally we should include dark energy perturbations (Park et al. 2009; Christopherson 2010), and these will be investigated in future work.

Since the non-adiabatic pressure perturbation is gauge-invariant it can be evaluated in any gauge, and we choose for numerical simplicity the synchronous gauge, in which the line-element takes the form

$$ds^2 = a^2(\eta) \left(-d\eta^2 + (\delta_{ij}(1 - \psi) + E_{,ij}) dx^i dx^j \right), \quad (4)$$

employing the conformal time η . The residual gauge freedom is removed by choosing to co-move with cold dark matter, i.e. $v_c(k, \eta) = 0$. The governing equations are then presented in Ma & Bertschinger (1995).¹

Taking the entropy perturbation between each species to vanish at some early epoch gives the fluids the initial conditions in tight-coupling and deep radiation domination (Ma & Bertschinger 1995)

$$\begin{aligned} \delta_\gamma &= \delta_\nu = \frac{4}{3}\delta_b = \frac{4}{3}\delta_c = -\frac{2}{3}Ck^2\eta_i^2, \\ v_\gamma &= v_b = -\frac{1}{18}Ck^2\eta_i^3, \quad v_\nu = \frac{23+4R_\nu}{15+4R_\nu}v_\gamma, \quad v_c \equiv 0. \end{aligned} \quad (5)$$

These are supplemented by initial conditions for ψ , E and the neutrino shear which we do not present here; for clarity we have also neglected small corrections due to the non-zero matter density even in the early times, although they are included in the code. These initial conditions are implemented at $z_i \approx 100,000$ to ensure that all linear modes are strongly superhorizon when the code begins. The constant $R_\nu = \rho_\nu/(\rho_\gamma + \rho_\nu)$ is the relative neutrino abundance.

The system of equations can now be integrated to find the fluid and metric perturbations at arbitrary time. The power spectrum of a perturbation $\delta(\mathbf{k})$ is given by

$$\begin{aligned} P_\delta(k, \eta) &= \langle \delta(\mathbf{k})\delta^*(\mathbf{k}') \rangle \\ &= \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k) |\delta(k)|^2 (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') \end{aligned} \quad (6)$$

where $\mathcal{P}_\zeta(k) = A_*(k/k_*)^{n_s-1}$ is defined using the curvature perturbation $\zeta \approx \mathcal{R}$ which is constant on superhorizon scales. The amplitude A_* at the pivot wavenumber k_* take the WMAP7 values $A_* = 2.42 \times 10^{-9}$ at $k_* = 0.002 h \text{Mpc}^{-1}$. Figure 1 shows the power spectrum of the baryon density contrast $\delta_b(k)$,

$$P_b(k, \eta) = \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k) |\delta_b(k)|^2, \quad (7)$$

for the range of wavenumbers and redshifts we sample. We note that the maximum plotted redshift $z_{\text{max}} \approx 50,000$ and is therefore significantly later than the initial redshift $z_i \approx 100,000$.

¹ In terms of the variables h and η_{MB} in Ma & Bertschinger (1995), the spatial curvature perturbations ψ and E (in Malik & Wands (2005)) are $\psi = \eta_{\text{MB}}$ and $E = -(h + 6\eta_{\text{MB}})/(2k^2)$, while the velocity perturbation is $v_\alpha = \theta_\alpha/k^2$.

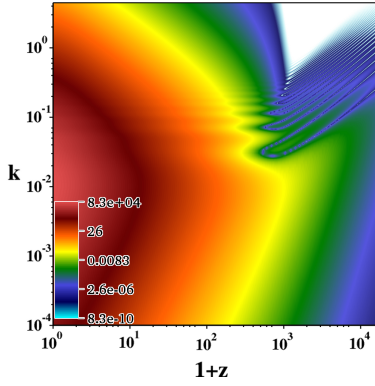


Figure 1. Power spectrum of the baryon density contrast, $P_b(k, \eta)$.

4 RESULTS

We can now derive an analytical approximation for the non-adiabatic pressure perturbation. Defining the scaled entropy differences between two fluids by

$$\Delta_{\alpha\beta} = (1 + w_\beta)\delta_\alpha - (1 + w_\alpha)\delta_\beta, \quad (8)$$

and using the fact that the density contrasts evolve as

$$\begin{aligned} \dot{\delta}_c + \frac{1}{2}\dot{h} &= 0, & \dot{\delta}_b + \frac{1}{2}\dot{h} + k^2 v_b &= 0, \\ \dot{\delta}_\gamma + \frac{2}{3}\dot{h} + \frac{4}{3}k^2 v_\gamma &= 0, & \dot{\delta}_\nu + \frac{2}{3}\dot{h} + \frac{4}{3}k^2 v_\nu &= 0, \end{aligned} \quad (9)$$

we find that the scaled entropy differences are governed by

$$\begin{aligned} \dot{\Delta}_{\gamma b} &= \frac{4}{3}k^2(v_b - v_\gamma), & \dot{\Delta}_{\gamma c} &= -\frac{4}{3}k^2 v_\gamma, \\ \dot{\Delta}_{\nu b} &= \frac{4}{3}k^2(v_b - v_\nu), & \dot{\Delta}_{\nu c} &= -\frac{4}{3}k^2 v_\nu. \end{aligned} \quad (10)$$

These equations are gauge-invariant, and apply at all times.

From Eq. (5) we see that on superhorizon scales, in radiation domination and during tight-coupling, the CDM terms will dominate. Furthermore, the present neutrino abundance implies $R_\nu \approx 0.4$ and so

$$\Delta_{\nu c} > \Delta_{\gamma c} \gg \Delta_{\nu b} \gg \Delta_{\gamma b}. \quad (11)$$

Assuming tight-coupling to be exact, implying $\Delta_{\gamma b} = 0$, integrating Eqs. (10) in the early universe then gives

$$\delta P_{\text{rel}} \approx \frac{3/216 \times H_0^2 \Omega_c (15 + 12R_\nu)}{8\pi G \left(1 + \frac{3}{4}a \frac{\Omega_M}{\Omega_R}\right) (15 + 4R_\nu)} C k^4 \eta^4 a^{-3} \propto k^4 a. \quad (12)$$

In finding this we have used that in radiation domination $a \propto \eta$. A rough approximation to the behaviour in early matter domination can be found by setting $a \propto \eta^2$, giving the rapidly decaying

$$\delta P_{\text{rel}} \propto k^4 a^{-2} (1 + 3\Omega_M a / 4\Omega_R)^{-1} \quad (13)$$

which suggests that δP_{rel} reaches a maximum at or around the epoch of matter/radiation equality. However, Eq. (13) applies only very close to equality as it assumes a similar growth of perturbations to that in radiation equality; the most we can say is that we expect the non-adiabatic pressure perturbation to decay early in matter domination. This will be confirmed by a more accurate, numerical analysis.

Given the primordial power spectrum $\mathcal{P}_\zeta(k) \propto k^{n_s-1}$ the power spectrum of the non-adiabatic pressure perturbation in the early universe is then

$$\langle \delta P_{\text{rel}}(\mathbf{k}) \delta P_{\text{rel}}^*(\mathbf{k}') \rangle \propto k^{4+n_s} a^2. \quad (14)$$

[h]

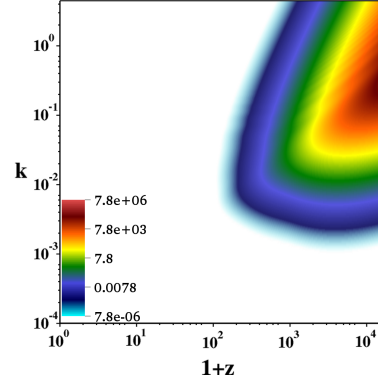


Figure 2. Power spectrum of the non-adiabatic pressure perturbation $P_{\delta P_{\text{rel}}}(k, \eta)$.

The non-adiabatic pressure perturbation on large scales and at early times grows with scale factor. It is also strongly dependent on k and decreases rapidly on superhorizon scales, which is to be expected.

To calculate the evolution of the non-adiabatic pressure perturbation we employ a modified version of the CMBFast code (Seljak & Zaldarriaga 1996; Bertschinger 1995) to recover the velocity perturbations for the full system from radiation domination to the present day, across a wide range of scales. The non-adiabatic pressure perturbation is then found by integrating Eqs. (10).

The power spectrum of the non-adiabatic pressure perturbation is plotted across scale and redshift in Figure 2. The oscillations before recombination are prominent, and δP_{rel} indeed peaks at approximately matter/radiation equality. It is orders of magnitude more significant on smaller scales than on larger scales and at early times peaks at approximately $k \sim 0.1 h/\text{Mpc}$. Across all wavenumbers δP_{rel} decays rapidly after matter/radiation equality. The non-adiabatic pressure perturbation is negative.

The left panel of Figure 3 shows the evolution of this non-adiabatic pressure perturbation for wavelengths between $k = 10^{-5} h/\text{Mpc}^{-1}$ and $k = 10^{-1} h/\text{Mpc}$. The growth with scale-factor at the earliest times is apparent, as is the decay during matter domination. However, the period at which δP_{rel} peaks is extended, being approximately within an order-of-magnitude of the peak between $z \approx 10,000$, significantly before matter/radiation equality, and recombination at $z \approx 1,100$, after which it decays rapidly. For very short wavelengths δP_{rel} peaks at earlier times and decays more rapidly. The decay after recombination goes as $P_{\delta P_{\text{rel}}} \propto a^{-6}$ for smaller scales and as $P_{\delta P_{\text{rel}}} \propto a^{-5}$ for larger scales, implying $\delta P_{\text{rel}} \propto a^{-3}$ and $\delta P_{\text{rel}} \propto a^{-5/2}$ on small and large scales respectively.

The right panel of Figure 3 shows the dependence of the non-adiabatic pressure perturbation on wavenumber for a range of redshifts from deep radiation domination to the present epoch. The decay towards the present epoch is clear, and the scale at which the perturbation has most power grows from $k \sim 0.2$ in deep radiation domination to $k \sim 0.02$ at a redshift of $z \approx 500$. The power spectrum scales as $P_{\delta P_{\text{rel}}} \propto k^{-5/2}$ on small scales and as $P_{\delta P_{\text{rel}}} \propto k^5$ on large scales. The non-adiabatic pressure perturbation itself then scales as $\delta P_{\text{rel}} \propto k^{1/4}$ on small scales and $\delta P_{\text{rel}} \propto k^4$ on large scales.

These results can be summarised as

$$\delta P_{\text{rel}} \propto a \begin{cases} k^4, & k \lesssim 10^{-1} h/\text{Mpc} \\ k^{1/4}, & k \gtrsim 10^{-1} h/\text{Mpc} \end{cases} \quad (15)$$

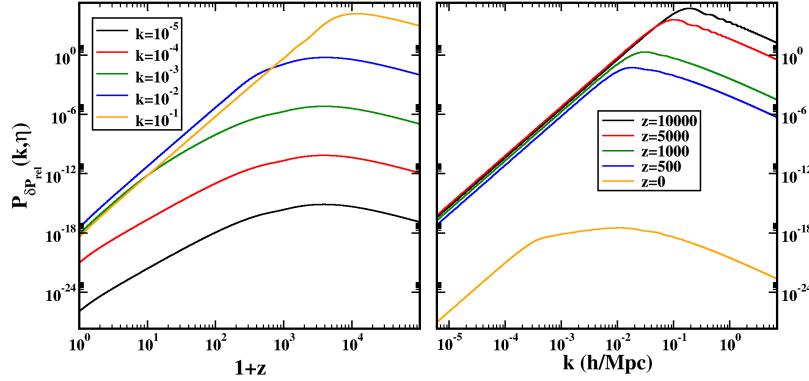


Figure 3. Power spectrum of the non-adiabatic pressure perturbation $P_{\delta P_{\text{rel}}(k, \eta)}$ as a function of redshift for set wavenumber (left); and as a function of wavenumber for set redshift (right).

at early times, and

$$\delta P_{\text{rel}} \propto \begin{cases} a^{-7/2} k^4, & k \lesssim 10^{-3} - 10^{-4} h/\text{Mpc} \\ a^{-3} k^{1/3}, & k \gtrsim 10^{-1} h/\text{Mpc} \end{cases} \quad (16)$$

at late times. This contrasts with Christopherson et al. (2011), which assumed $\delta P_{\text{nad}} \propto k^\alpha a^{-5}$ with $\alpha \neq 1$ before setting $\alpha = 2$. This choice underestimates the gradient and overestimates the rate of decay of δP_{nad} on large scales while failing to model the behaviour on small scales. We would like to emphasise that we are working with small but finite wavenumbers; in the limit $k \rightarrow 0$ the non-adiabaticity vanishes, and the rapid decay ($\sim k^4$) on super-horizon scales is very clear.

The contribution to the non-adiabatic pressure perturbation due to the entropy difference between fluid species α and β is

$$\delta P_{\text{rel}, \alpha\beta} = \frac{\rho_\alpha \rho_\beta ((1 + w_\beta) \delta_\alpha - (1 + w_\alpha) \delta_\beta)}{3(\rho + p)}. \quad (17)$$

Figure 4 shows the contributions of these terms at wavenumbers $k = 10^{-4} h\text{Mpc}^{-1}$ and $k = 10^{-1} h\text{Mpc}^{-1}$. At early times, the CDM/neutrino contribution dominates slightly over the CDM/photon contribution, as was expected in Eq. (11). The baryon/neutrino term is significantly smaller, and the baryon/photon term (as expected) is smaller again.

After recombination, this picture changes. The decoupling of the photons and baryons causes their relative entropy to increase significantly until at $z \approx 100$, $\delta P_{\text{rel}, \gamma b} \gtrsim \delta P_{\text{rel}, \nu b}$. At late times the baryon perturbations and the CDM perturbations become approximately equal, while v_ν and v_γ behave similarly to one-another. It is no surprise that the CDM/neutrino and CDM/photon contributions exhibit the same behaviour as the baryon/neutrino and baryon/photon terms, and so after recombination, $\delta P_{\text{rel}, \gamma c} > \delta P_{\text{rel}, \nu c}$. The difference between the CDM/radiation and baryon/radiation terms is then due to the relative abundance of CDM over baryons; in the concordance model, the CDM terms dominate by a factor of 4 over the baryon terms.

On small scales the behaviour is similar to the large-scale behaviour, except that $\delta P_{\text{rel}, \gamma c} \gtrsim \delta P_{\text{rel}, \nu c}$ for the entire period sampled. For $z \gtrsim 5 \times 10^4$ this would be expected to be reversed. For all times and across all linear scales, one can therefore find a reasonable approximation to the non-adiabatic density perturbation by considering only the relative entropies between the radiative species and the cold dark matter.

5 DISCUSSION

We have calculated the non-adiabatic pressure perturbation in the WMAP7 concordance cosmology, finding both its magnitude and its scaling properties. It is non-vanishing and grows in the early universe, becoming most significant at around matter/radiation equality. For a wide range of wavenumbers it remains within an order of magnitude of its peak until recombination after which it decays rapidly.

Although the existence of the non-adiabatic pressure perturbation does not influence existing *first-order* CMB calculations employing Boltzmann codes such as CAMB or CMBFast, its study is nevertheless extremely important. Firstly, this quantity is not actually calculated in Boltzmann codes and yet has a concrete physical meaning – it is directly related to the entropy perturbation. One example we used to motivate its importance is the vorticity it will produce at second-order, the study of which is ongoing, and the magnitude of which could be sufficient as to provide an additional foreground to the CMB polarisation signals. In turn this may produce magnetic fields that could influence recombination physics or provide seed fields for the observed cluster magnetic fields, or to influence large-scale structure formation in the more recent universe. Secondly, even though the non-adiabatic pressure is formed of linear combinations of quantities evaluated in Boltzmann codes, it cannot be accurately recovered by naively combining these quantities. We have assumed entirely adiabatic initial conditions. On the largest scales and at early times, then, we have for instance $\delta_\gamma \approx (4/3)\delta_b$. The non-adiabatic pressure depends on the difference between these two quantities, which is extremely small. Calculating this numerically is then rather unstable; naively reconstructing the non-adiabatic pressure simply from the perturbations in one of the Boltzmann codes could give entirely the wrong behaviour on very large scales and at very early times.² As such, one needs to modify the codes to evaluate the entropy differences directly, as we have in this work.

This perturbation will have various effects. Whereas the curvature perturbation is conserved on large scales at early times while the universe can be modelled by a single barotropic fluid, this is no longer case at later times when the universe has to be modelled as a multi-fluid system. Another effect is the generation of vorticity

² We found an extreme case where the reconstructed non-adiabatic pressure *decays* at early times, since the rounding error in $\delta_\gamma - (4/3)\delta_b$ grows ever more dominant for increasing redshifts.

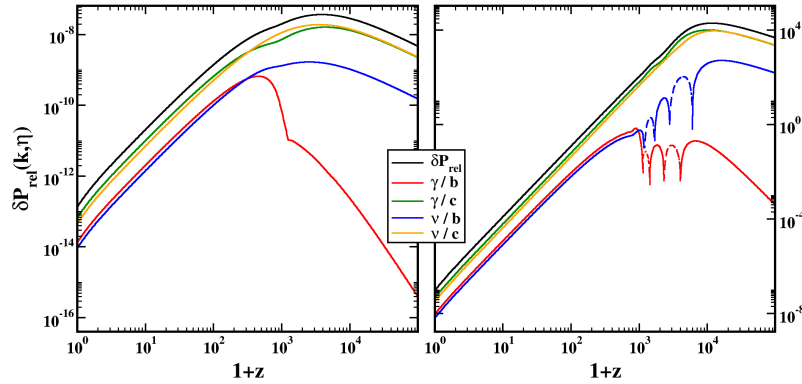


Figure 4. The contributions to the total relative non-adiabatic pressure perturbation arising from the photon/baryon, photon/CDM, neutrino/baryon and neutrino/CDM couplings at $k = 10^{-4} h\text{Mpc}^{-1}$ (left) and $k = 10^{-1} h\text{Mpc}^{-1}$ (right). Absolute values are plotted; the terms are all negative at late epochs.

at second order. The assumptions in Christopherson et al. (2011) were conservative; with the scalings found in our numerical study, we might expect the resulting vorticity to be enhanced. This will form the focus of a future study. In summary, we have shown that the non-adiabatic pressure perturbation is non-zero in the early universe for generic initial conditions and matter content, even if the intrinsic entropy perturbations are negligible.

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Wands D., Malik K. A., Lyth D. H., Liddle A. R., 2000, Phys. Rev., D62, 043527

REFERENCES

- Abazajian K. N., et al., 2009, *Astrophys. J. Suppl.*, 182, 543
Ade P. A. R., et al., 2011, *Astron. Astrophys.*, 536, A1
Bertschinger E., 1995
Christopherson A. J., 2010, *Phys. Rev.*, D82, 083515
Christopherson A. J., Malik K. A., Matravets D. R., 2009, *Phys. Rev.*, D79, 123523
Christopherson A. J., Malik K. A., Matravets D. R., 2011, *Phys. Rev.*, D83, 123512
Doran M., 2005, *JCAP*, 0510, 011
Kodama H., Sasaki M., 1984, *Prog. Theor. Phys. Suppl.*, 78, 1
Komatsu E., et al., 2011a, *Astrophys.J.Suppl.*, 192, 18
Komatsu E., et al., 2011b, *Astrophys.J.Suppl.*, 192, 18
Lesgourgues J., 2011
Lewis A., Challinor A., Lasenby A., 2000, *Astrophys. J.*, 538, 473
Li H., Liu J., Xia J.-Q., Cai Y.-F., 2011, *Phys. Rev.*, D83, 123517
Liddle A. R., Lyth D. H., 2000, *Cosmological inflation and large-scale structure*. CUP, Cambridge, UK
Ma C.-P., Bertschinger E., 1995, *Astrophys.J.*, 455, 7
Malik K. A., 2005, *JCAP*, 0511, 005
Malik K. A., Wands D., 2005, *JCAP*, 0502, 007
Mangilli A., Verde L., Beltran M., 2010, *JCAP*, 1010, 009
Park C.-G., Hwang J.-c., Lee J.-h., Noh H., 2009, *Phys. Rev. Lett.*, 103, 151303
Seljak U., Zaldarriaga M., 1996, *Astrophys. J.*, 469, 437
Valliviita J., Giannantonio T., 2009, *Phys. Rev.*, D80, 123516